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THE ANALYTICS OF THE WAGE EFFECT OF IMMIGRATION

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ABSTRACT

The theory of factor demand has important implications for the study of the impact of immigration on wages in both sending and receiving countries. This paper examines the implications of the theory in the context of a model of a competitive labor market where the wage impact of immigration is influenced by such factors as the elasticity of product demand, the rate at which the consumer base expands as immigrants enter the country, the elasticity of supply of capital, and the elasticity of substitution across inputs of production. The analysis reveals that the short-run wage effect of immigration is negative in a wide array of possible scenarios, and that even the long run effect of immigration may be negative if the impact of immigration on the potential size of the consumer base is smaller than its impact on the size of the workforce. The closed-form solutions permit numerical back-of-the-envelope calculations of the wage elasticity. The constraints imposed by the theory can be used to check the plausibility of the many contradictory claims that appear throughout the immigration literature.

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The Analytics of the Wage Effect of Immigration

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I. Introduction

The resurgence of large-scale immigration motivated the development of a large literature that examines what happens to the labor market in both receiving and sending countries as a result of an immigration-induced change in supply. The textbook model of a competitive labor market has clear and unambiguous implications about how wages should adjust to an immigrant-induced labor supply shift. In the short run, higher levels of immigration should lower the wage of competing workers and increase the wage of complementary workers. Despite the common-sense intuition behind these predictions, the empirical literature seems to be full of contradictory results. Some studies claim that immigration has a substantial impact on wages in receiving and sending countries (e.g., Borjas, 2003; Mishra, 2007), while other studies claim the impact is negligible (Card, 2005; Ottaviano and Peri, 2008).

This paper takes a step back from the empirical debate and asks a simple question: What does factor demand theory have to say about the potential wage impact of immigration-induced supply shifts? Since Marshall's time, economists have had a good understanding of the factors that generate elastic or inelastic labor demand curves, and how the elasticity of labor demand is affected by substitution and scale effects.[1]

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[1] Hicks (1932) gives the classic presentation of Marshall's rules of derived demand. Ewerhart (2003) and Kennan (1998) provide much simpler and clearer derivations.

Unfortunately, much of the empirical literature on the wage impact of immigration (particularly in the 1990s) disregarded practically all of these theoretical insights, and instead took a data-mining approach: running regressions or difference-in-differences models to examine if the wage evolution in labor markets most affected by immigration differed from that observed in other markets. Few of these studies were guided or informed by the implications of factor demand theory.

More recently, beginning with Card (2000) and Borjas (2003), the literature has taken a turn and begun to pay much closer attention to the underlying economics of the problem.[2] This paper presents a model of factor demand that allows for the wage effect of immigration to work through the channels first delineated in Marshall's rules of derived demand. As with Marshall's rules, the resulting model has much to say about the factors that are likely to lead to small or large wage effects from immigration.

The analysis makes two contributions. First, the model generates a closed-form solution of the wage effect of immigration, allowing us to easily calculate back-of-the-envelope estimates of the wage effect of immigration under a large number of potential scenarios. Put differently, factor demand theory imposes severe constraints on the potential sign and numerical value of these wage effects.

Second, the model cleanly demonstrates the important difference in studying the impact of immigration on the wage level as opposed to studying the impact of immigration on the wage distribution. It turns out that *completely different* parameters determine the two sets of results. The impact of immigration on the wage level in the sending or receiving

[2] For instance, a recent study by Wagner (2009) is the first to attempt to isolate empirically the substitution and scale effects of an immigration-induced supply shift.

country depends on the set of parameters first enunciated by Marshall as determining the elasticity of labor demand: the extent of substitution between labor and capital, the elasticity of product demand, the elasticity of supply of capital, and labor's share of income.[3] The impact of immigration on the wage distribution, however, does not depend on *any* of these factors, but instead depends on how different skill groups interact in production and on the skill composition of immigrants relative to that of natives. The "separability" of these results again allows simple back-of-the-envelope calculations that suggest the possible range of estimates for the distributional impact of immigration.

II. Homogeneous Labor in a One-Good Cobb-Douglas Economy

It is instructive to begin with the simplest model of the labor market: a single aggregate good, Q, is produced using a Cobb-Douglas production function that combines a homogeneous labor input (L) and capital (K), hence $Q = K^{\alpha} L^{1-\alpha}$. It turns out that the key insights provided by this model—including a range of numerical values for the wage effect of immigration—carry through to more complicated models that allow for a generalized production technology or for heterogeneous labor, as well as for various feedback effects.

To fix ideas, consider first a model where the product price p is fixed—one can think of p as the numeraire or perhaps it is set in the global market. In a competitive market, each input will be paid its value of marginal product:

(1a) $$w = p(1-\alpha) K^{\alpha} L^{-\alpha},$$

[3] The immigration context adds another important parameter to this standard framework: the extent to which immigrants differentially increase the size of the consumer base and the size of the workforce.

(1b) $$r = p\alpha K^{\alpha-1} L^{1-\alpha},$$

where w is the wage rate and r is the price of capital.

In this simple framework, the parameter α gives capital's share of income, or s_K. It is useful to consider two situations: the short run and the long run. By definition, the capital stock is fixed in the short run and the price of capital is fixed in the long run. Suppose an immigrant influx increases the size of the workforce. By differentiating equations (1a) and (1b), it is easy to show that:[4]

(2a) $$\left.\frac{d\log w}{d\log L}\right|_{dK=0} = -s_K.$$

(2b) $$\left.\frac{d\log w}{d\log L}\right|_{dr=0} = 0.$$

Throughout this paper, the term $\frac{d\log w}{d\log L}$ will be called the "wage elasticity" of immigration. The Cobb-Douglas technology not only provides a simple expression for this wage elasticity, but suggests a range of numerical values as well. It is well known that labor's share of income in the United States ($s_L = 1 - s_K$) has hovered around 0.7 for many decades. This implies that the short-run wage elasticity is -0.3. In the long run, the capital stock adjusts fully and the wage elasticity exactly equals 0.0. In other words, to the extent that the Cobb-Douglas technology is a reasonable approximation of the aggregate labor

[4] The labor supply of the pre-existing workforce is assumed to be inelastic. The immigration-induced supply shift can then be represented as an outward shift of an inelastic supply curve.

market, one would expect the wage elasticity to lie between 0.0 and -0.3, depending on the extent to which capital has adjusted to the presence of the immigrant influx.

It is worth emphasizing that the absence of a wage effect from immigration in the long run is a general property of any model with a constant returns to scale (CRS) production function. A CRS production function implies that input prices depend only on the capital/labor ratio. The long-run assumption that the price of capital is constant is effectively building in the restriction that the capital/labor ratio is also constant. If immigrants increase the size of the workforce by 10 percent, the capital stock must eventually also increase by 10 percent. In the end, the wage returns to its pre-immigration level.

III. A Two-Good Economy with Homogeneous Labor

I now expand the basic Cobb-Douglas model in several ways. First, I assume that there are two goods in the economy; one good is produced domestically, while the other good is imported. Second, I relax the assumption that the elasticity of substitution between labor and capital in the production of the domestic good is unity. Third, I allow for the possibility of changes in product demand both because immigration may have changed the price of the domestically produced product (encouraging consumers to change their quantity demanded) and because immigrants themselves will consume the product. Finally, I explicitly introduce a supply curve of domestic capital. The resulting model has much in common with derivations of Marshall’s rules of derived demand. The technical details are summarized in the mathematical appendix.

There are two goods consumed in a large economy.[5] The good q is produced domestically, and the good y is imported.[6] To fix ideas, I initially assume that the price of the imported good y is set in the global marketplace (or, alternatively, that it is produced at constant marginal cost). In this context, the price of y is the numeraire and set to unity. I will relax this assumption below and introduce an upward-sloping foreign export supply curve for y.

Each person j in the domestic labor market has a quasilinear utility function given by:

(3) $$U(y,q) = y + g_j^* q^{\xi},$$

where the weight g^* reflects the consumer's relative preference for the domestic good and may be different for different consumers (or different groups of consumers). The utility function will be quasiconcave only if $0 < \xi < 1$. As I show below, this restriction has implications for the magnitude of the price elasticity of demand. Let Z be the consumer's income. The budget constraint is then given by:

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[5] The introduction of a second good into the model is crucial if one wishes to examine how immigration affects aggregate product demand and prices. If there were only one good in the economy, all units of that good are sold regardless of the price. The framework developed below is related to the standard 2×2×2 model in international trade theory (Dixit and Norman, 1980).

[6] This approach, of course, implies that immigration and trade are complements since there is complete specialization of goods production. If immigration and trade were substitutes, as in Mundell's (1957) classic analysis, there may then be factor price equalization across countries. Immigration would have no wage effects and would only alter the distribution of outputs as described by the Rybczynski Theorem. I do not address the long-running debate over whether immigration and trade are complements or substitutes. The model presented below is instead designed to depict an economic environment where wage differences exist and induce labor to migrate internationally.

(4) $Z = y + pq$.

Utility maximization implies that the product demand function for the domestic good is:

(5) $$q_j = g_j\, p^{-1/(1-\xi)},$$

where q_j is the amount of the good consumed by consumer j; and g_j is the rescaled person-specific weight. The quasilinear functional form for the utility function implies that the consumer's demand for the domestic product does not depend on his income. The assumption that there are no wealth effects will also be relaxed below.

Three types of persons consume good q: domestic workers, domestic capitalists, and consumers in other countries. Let C_L be the number of domestic workers, C_K be the number of domestic capitalists, and C_X be the number of consumers in the "rest of the world."[7] I assume that each type of consumer has the same quasilinear utility function in (3), but that the weighting factor g may differ between domestic and foreign consumers. The total quantity demanded by domestic consumers (Q_D) and foreign consumers (Q_X) is then given by:

(6a) $$Q_D = g_D (C_L + C_K) p^{-1/(1-\xi)},$$

(6b) $$Q_X = g_X C_X p^{-1/(1-\xi)}.$$

[7] Since there are no wealth effects, it is not necessary to specify what income Z is for each of the groups. I will give a precise definition when I introduce wealth effects below.

Balanced trade requires that expenditures on the imported good *y* equal the value of the exports of good *q*:

(7) $$wL + rK - g_D(C_L + C_K)p^{-\xi/(1-\xi)} = g_X C_X p^{-\xi/(1-\xi)},$$

where (wL + rK) gives the total payment to domestic factors of production L and K. In a competitive market, the payment to each factor of production equals its value of marginal product. If the production function is linear homogeneous, Euler's theorem implies that the expression in (7) can be rewritten as:

(8) $$wL + rK = p(Q_L L + Q_K K) = pQ = \left[g_D(C_L + C_K) + g_X C_X\right]p^{-\xi/(1-\xi)}.$$

where Q_i is the marginal product of factor *i*. It follows that aggregate market demand for the domestic good is given by:

(9) $$Q = C\,p^{-1/(1-\xi)},$$

where $C = g_D(C_L + C_K) + g_x C_X$, the (weighted) number of consumers. Note that the demand elasticity ($d \log Q/d \log p$) is greater than unity (in absolute value). This is an important implication of quasilinear utility functions, and the restriction will be used to interpret some of the results presented below.

A crucial question in evaluating the impact of immigration on the domestic labor market is: How does an immigration-induced increase in the size of the workforce affect the size of the consumer base?[8] Let $C(L)$ be the function that relates the number of consumers to the number of workers, and let $\phi = d \log C/d \log L$. An important special case occurs when the elasticity $\phi = 1$, so that the immigrant influx leads to a proportionately equal increase in the (weighted) number of consumers and the number of workers. I will refer to the assumption that $\phi = 1$ as the case of *product market neutrality*. The "neutrality," of course, refers to the fact that the immigration-induced supply shift has the same relative impact on the size of the consumer base and the size of the workforce.

Note that it is easy to account for different product demand preferences between immigrants and natives by allowing for non-neutrality, i.e., by allowing for deviations from unity in the elasticity ϕ. For instance, if immigrants tend to prefer the consumption of the imported good, an immigrant influx that increases the size of the workforce by x percent would likely lead to a smaller percent increase in the number of "effective" consumers for the domestic good.

Equation (9) suggests that an immigration-induced supply shift will have two distinct effects in the domestic labor market through product demand: First, the price of the domestic good might change, moving current consumers along the existing product

[8] Even though the answer to this question plays an important role in determining the wage impact of immigration, it has not received any attention in the empirical literature.

demand curve; second, because immigrants are themselves "new" consumers, the market product demand curve will shift out and the magnitude of this shift will depend on ϕ.[9]

It is analytically convenient to work out the model in terms of the inverse product demand function:

(10) $$p = C^{\eta} Q^{-\eta},$$

where η is the inverse price elasticity of demand, with $\eta = 1 - \xi \geq 0$. Note again that the quasi-linear utility function restricts the inverse elasticity of demand η to be smaller than 1.

The production technology for the domestic product is given by the CES production function:

(11) $$Q = [\alpha K^{\delta} + (1-\alpha) L^{\delta}]^{1/\delta},$$

where $\delta \leq 1$. The elasticity of substitution between labor and capital is $\sigma = 1/(1 - \delta)$. Note that the production function in (11) maintains the CRS assumption from the previous section.

Finally, the supply of domestic capital is given by the supply function:

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[9] The issue of whether immigrants are "new" consumers can be approached in a number of ways. It could be argued, for instance, that immigration substantially raises the number of domestic consumers (through the increase in C_L), but leads only to a trivial decline in the relative number of consumers from abroad. Alternatively, it may be that immigrants change their preferences for the domestic good once they reside in their new home. Even though the increase in C_L may be completely offset by the decline in C_X, the weight determining the post-migration demand of the immigrants for the domestic good increases from g_X to g_D.

(12) $$r = K^{\lambda},$$

where $\lambda \geq 0$, and is the inverse elasticity of supply of capital. The two special cases introduced in the previous section for the short run and the long run correspond to $\lambda = \infty$ and $\lambda = 0$, respectively.

In a competitive market, input prices equal the value of marginal product:

(13a) $$w = (1-\alpha) C^{\eta} Q^{1-\delta-\eta} L^{\delta-1}.$$

(13b) $$r = \alpha\, C^{\eta} Q^{1-\delta-\eta} K^{\delta-1}.$$

Let $d \log L$ represent the immigration-induced percent change in the size of the workforce. By differentiating equations (13a) and (13b), allowing for the fact that the supply of capital is given by equation (12), it can be shown that:

(14) $$\frac{d\log w}{d\log L} = \frac{-\lambda(1-\delta-\eta)s_K}{(1+\lambda-\delta)-(1-\delta-\eta)s_K} - \frac{(1+\lambda-\delta)\eta(1-\phi)}{(1+\lambda-\delta)-(1-\delta-\eta)s_K}.$$

Consider initially the special case of product market neutrality (i.e., $\phi = 1$), so that immigration expands the size of the consumer pool by the same proportion as its expansion of the workforce. The wage elasticity then reduces to:

(14a) $$\left.\frac{d\log w}{d\log L}\right|_{\phi=1} = \frac{-\lambda(1-\delta-\eta)s_K}{(1+\lambda-\delta)-(1-\delta-\eta)s_K}.$$

Note that in the long run ($\lambda = 0$), the wage elasticity goes to zero. Note also that the denominator of equation (14a) is unambiguously positive.[10] As long as there is incomplete capital adjustment ($\lambda > 0$), therefore, the wage elasticity will be negative if $(1 - \delta - \eta) > 0$. Define η^* to be the elasticity of product demand (i.e., $\eta^* = 1/\eta$). It is then easy to show that $(1 - \delta - \eta) > 0$ implies that:

$$\eta^* > \sigma. \tag{15}$$

In other words, even after allowing for a full response by *all* consumers in the product market, the wage effect of immigration will be negative if there is incomplete capital adjustment and if it is easier for consumers to substitute among the available goods than it is for producers to substitute between labor and capital. This latter condition, of course, has a familiar ring in labor economics—as it happens to be identical to the condition that validates Marshall's second rule of derived demand: An increase in labor's share of income leads to more elastic demand "only when the consumer can substitute more easily than the entrepreneur" (Hicks, 1932, p. 246).

It turns out, however, that the condition in equation (15) can be independently corroborated within the context of the immigration model presented in this section. In particular, recall that the quasilinear utility function used to derive a consumer's product demand function restricted the inverse of the elasticity of product demand to be less than

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[10] In particular, $(1 + \lambda - \delta) - (1 - \delta - \eta)s_K = \lambda + (1 - \delta)s_L + \eta s_K \geq 0$. The denominator is strictly positive if $\lambda > 0$, or $\delta < 1$, or $\eta > 0$.

unity, hence $\eta^* > 1$. Equation (15), therefore, is satisfied for the Cobb-Douglas production function, as well as for any production function that allows less substitution between labor and capital than the Cobb-Douglas. Hamermesh's (1993) survey of labor demand concludes that the Cobb-Douglas is a reasonably good approximation to the aggregate production function in the United States and, if anything, the actual estimates of σ may be slightly lower than 1.0.

The restriction that $(1 - \delta - \eta) > 0$ is positive can also be derived as a second-order condition to the problem faced by a social planner trying to determine the optimal amount of immigration in the context of the current model. One important feature of the competitive market model presented in this section is that the wage-setting rule ignores the fact that an additional immigrant affects product demand, so that the marginal revenue product of an immigrant is not equal to his value of marginal product. Suppose a social planner internalizes this externality and wishes to admit the immigrant influx that maximizes gross *domestic* product net of any costs imposed by immigration.[11] More precisely, the social planner wishes to maximize:

(16) $$\Omega = pQ - Mh = C^{\eta} Q^{1-\eta} - Mh,$$

where M gives the number of immigrants and h gives the (constant) cost of admitting an additional immigrant (perhaps in terms of providing social services, etc.). For simplicity,

[11] Benhabib (1996) presents a detailed analysis of the choice of an optimal immigration policy in the context of a political economy model.

consider the case with product market neutrality. In the mathematical appendix, I show that the second-order conditions for this maximization problem are satisfied if:[12]

(17) $(1 - \eta) > 0$ and $(1 - \delta - \eta) > 0$.

In short, as long as the social planner takes into account that the marginal revenue produced by an additional immigrant is not constant, the wage elasticity in equation (14a) must be negative. Put differently, the scale effect resulting from immigration—regardless of whether it occurs through an expansion of the capital stock or through an expansion in product demand—can never be sufficiently strong to lead to a wage increase. And, in fact, as long as capital adjustment is incomplete, the wage effect must be negative.

A vast number of special cases in the presence of product market neutrality can be obtained by evaluating equation (14a) at specific values of the four parameters that determine the wage effect of immigration (i.e., λ, η, δ, and s_L). For instance, it is easy to calculate the wage effect in the simple Cobb-Douglas economy considered in the previous section—simply evaluate (14a) after setting $\delta = 0$, $\lambda = \infty$, $\eta = 0$ for the short run, and $\delta = 0$, $\lambda = 0$, and $\eta = 0$ for the long run.

It is equally easy to measure the size of the scale effect triggered by immigration by considering the simple case of a Cobb-Douglas economy in the short run. The wage elasticity in (14a) then collapses to:

[12] The second order conditions would also be satisfied if per-immigrant cost *h* was not constant, but increased with the number of immigrants. Note that the restriction that $\eta < 1$ also has a familiar ring since it is a well-known feature of monopoly models. In that context, the monopolist internalizes the externality that additional units of the good lower the price. As a result, it produces at a point where product demand is elastic.

(18) $$\left.\frac{d\log w}{d\log L}\right|_{\substack{\phi=1,\\ \delta=0,\\ \lambda=\infty.}} = -(1-\eta)s_K\ .$$

By contrasting this elasticity with the analogous effect in the one-good model presented in equation (2a), it is easy to see that the scale effect of immigration equals ηs_K. In the absence of the scale effect, the wage elasticity would equal -0.3. If the inverse elasticity of product demand is 0.5 (implying a product demand elasticity of 2.0), the wage elasticity would fall to -0.15. In other words, the short-run adverse effect of immigration on the wage can be greatly alleviated through increased product demand—as long as the product demand elasticity is sizable.

It is important to emphasize that this negative wage impact can persist—even in the long run—if the product market neutrality assumption does not hold. Consider, in particular, the non-neutral case where immigration does not expand the size of the consumer base as rapidly as it expands the size of the workforce (i.e., $\phi < 1$). The second term in (14) is then negative and does not vanish as λ goes to zero. In other words, the adverse wage effect of immigration is larger because there are "too many" workers and "too few" consumers. This result has interesting implications for the economic effect of immigration when immigrants send a large fraction of their earnings to the sending country in the form of remittances. The negative effect of remittances on wages in the receiving country is permanent; it does not disappear even after capital has fully adjusted to the immigrant influx.[13]

Moreover, the wage consequences of even slight deviations from product market neutrality can be numerically sizable. As an illustration, consider the long run effects in a Cobb-Douglas economy. In the long run, the first term in equation (14) vanishes and the wage elasticity is given by:

$$\left.\frac{d\log w}{d\log L}\right|_{\substack{\lambda=0\\ \delta=0}} = \frac{-\eta(1-\phi)}{1-(1-\eta)\,s_K}. \tag{19}$$

Suppose that $\phi = 0.90$, so that an immigration-induced doubling of the workforce increases the size of the consumer pool by 90 percent. Suppose again that the inverse elasticity of product demand η is 0.5. Equation (19) then predicts that the *long-run* wage elasticity of immigration will equal -0.06.

Immigration and Prices

The wage effect summarized in equation (14) gives the wage impact of immigration in terms of the price of the imported product (i.e., the numeraire). It is also of interest to determine the impact of immigration relative to the price of the domestically produced good. After all, immigration has domestic product price effects both because the wage

[13] This conclusion, of course, requires that remittances continue indefinitely so that $\phi < 1$ regardless of how long immigrants have been in the receiving country.

drops and because immigrants themselves shift the product demand curve outwards.[14] By differentiating equation (10) with respect to the immigration-induced supply shift, it can be shown that the effect of immigration on the domestic price is given by:

$$\text{(20)} \qquad \frac{d\log p}{d\log L} = \frac{\lambda\eta s_K}{(1+\lambda-\delta)-(1-\delta-\eta)s_K} - \frac{\eta(1-\phi)[\lambda+(1-\delta)s_L]}{(1+\lambda-\delta)-(1-\delta-\eta)s_K}.$$

Suppose that there is product market neutrality. The second term of (20) then drops out. Immigration has no price effect either in the long run ($\lambda = 0$) or if the immigrant influx has no impact on the domestic product market ($\eta = 0$). However, equation (20) shows that immigration *must* increase prices as long as the product demand curve is downward sloping and capital has not fully adjusted.[15]

The prediction that domestic prices rise at the same time that wages fall seems counterintuitive. However, it is easy to understand the economic factors underlying this result by simply differentiating the market product demand curve:

$$\text{(21)} \qquad \frac{d\log p}{d\log L} = \eta s_K\left(1-\frac{d\log K}{d\log L}\right) - \eta(1-\phi).$$

As long as there is product market neutrality, the price of the domestic good must rise whenever capital adjusts by less than the immigration-induced percent shift in supply. The

[14] Recent studies of the price effect of immigration include Lach (2007), Cortes (2008), and Saiz (2007).

[15] Of course, the inflationary effect of immigration is attenuated if $\phi < 1$ and product demand does not rise proportionately with the size of the immigrant influx.

intuition is clear: In the absence of full capital adjustment, the immigration-induced increase in domestic product demand cannot be easily met by the existing mix of inputs, raising the price of the domestic product.[16]

An important question, of course, is: what happens to the *real* wage defined in terms of the price of the domestic product (or w/p)? By combining results from equations (14) and (20), it is easy to show that the *real* wage elasticity of immigration (defined as the wage effect relative to the change in domestic prices) is given by:[17]

(22) $$\dot{w} = \frac{d\log(w/p)}{d\log L} = \frac{-\lambda(1-\delta)s_K}{(1+\lambda-\delta)-(1-\delta-\eta)s_K} - \frac{\eta(1-\phi)(1-\delta)s_K}{(1+\lambda-\delta)-(1-\delta-\eta)s_K}.$$

Note that if the product market neutrality assumption holds, the second term in (22) vanishes and immigration *must* reduce the real wage as long as capital does not fully adjust. Note also that this result does not depend on the relative magnitudes of the elasticities of

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[16] The response of the capital stock to the immigrant influx is given by:

$$\frac{d\log K}{d\log L} = \frac{(1-\delta)-(1-\delta-\eta)s_K}{(1+\lambda-\delta)-(1-\delta-\eta)s_K} - \frac{\eta(1-\phi)}{(1+\lambda-\delta)-(1-\delta-\eta)s_K}.$$

Note that if the product market neutrality assumption holds, the percent shift in the capital stock will be a fraction (between 0 and 1) of the immigration-induced percent shift in the size of the workforce.

[17] There are alternative ways of defining the real wage. For instance, one can define a price index $\overline{p} = p_y^{1-\mu_D} p^{\mu_D}$, where p_y Is the price of good y (which is, of course, fixed at 1), and μ_D is the share of income that is spent in good *y* in the domestic economy. Holding constant the share μ_D, it is easy to show that the resulting real wage elasticity is:

$$\frac{d\log(w/\overline{p})}{d\log L} = \frac{-\lambda[1-\delta-\eta(1-\mu_D)]s_K}{(1+\lambda-\delta)-(1-\delta-\eta)s_K},$$

which must be negative if the second-order conditions in equation (17) hold.

substitution and product demand. The negative impact of immigration on the real wage is not surprising. After all, immigration reduces the nominal wage and increases the price level simultaneously. To simplify the discussion, I will refer to the elasticity in (22) as the *real* wage elasticity of immigration.

In order to get a sense of the magnitude of the real wage elasticity, it is again instructive to refer back to the simplest example: a Cobb-Douglas economy in the short run. If there is product market neutrality, it is easy to show that:

$$\text{(23)} \qquad \left.\frac{d\log(w/p)}{d\log L}\right|_{\substack{\delta=0\\ \phi=1\\ \lambda=\infty}} = -s_K .$$

The short-run real wage elasticity is identical to that implied by the simplest one-good Cobb-Douglas model in equation (2a). Even after the model accounts for the fact that immigrants increase the size of the consumption base proportionately and that immigration-induced price changes move the pre-existing consumers along their product demand curve, the short-run wage elasticity is -0.3.

The theory of factor demand, therefore, clarifies an important misunderstanding: the often-heard argument that the outward shift in product demand induced by immigration will somehow return the economy to its pre-immigration equilibrium does not have any theoretical support. Instead, factor demand theory reveals that immigration will inevitably have an adverse effect on the real wage. Put differently, the number of domestically produced widgets that the typical worker in the receiving country can potentially buy will decline as the result of the immigrant influx—even after one accounts for the fact that immigrants themselves will increase the demand for widgets. And, under some conditions, the decline in the number of widgets that can be purchased is exactly the same as the decline found in the simplest factor demand model that ignores the role of immigrants in the widget product market.

Marshall's Rules Redux

As with Marshall's rules of derived demand, a great deal of insight into the underlying economics can be obtained by differentiating the real wage elasticity in equation (22) with respect to the parameters that determine its value.[18] To fix ideas, I focus on the case of product market neutrality. It is also useful to conduct this exercise in terms of the actual elasticity of substitution, the elasticity of product demand, and the elasticity of supply of capital (rather than some transformation or inverse of the relevant elasticity). In particular, note that $\sigma = 1/(1- \delta)$, and define the price elasticity of demand as $\eta^* = 1/\eta$; and the elasticity of supply of capital as $\lambda^* = 1/\lambda$. Since the long-run impact of immigration on the real wage is numerically equal to zero, I focus on the case of $\lambda^* < \infty$. It can then be shown that:[19]

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[18] Some caution is required when interpreting these derivatives. Labor's share of income is not constant unless the production function is Cobb-Douglas. The partial derivatives reported in equation (24) ignore the feedback effects that occur through changes in s_L. See Pemberton (1989) for a detailed discussion of the analogous (and universally ignored) issue in the derivation of Marshall's rules of derived demand.

[19] The easiest way to prove the rules is to convert the wage elasticities defined either in equation (14) or (22) into formulas that depend on the actual values of the elasticities rather than on their transformation. For example, in terms of the primitive parameters, equation (22) can be written as:

$$\dot{w} = \frac{-[\eta^* + \lambda^*(1-\phi)]s_K}{\eta^*(\lambda^* + \sigma) - \lambda^*(\eta^* - \sigma)s_K}.$$

The rules are easily obtained by partially differentiating this expression (but, again, note the important caveat in note 16).

(24) $$\frac{\partial \dot{w}}{\partial \sigma} > 0, \quad \frac{\partial \dot{w}}{\partial s_L} > 0, \quad \frac{\partial \dot{w}}{\partial \lambda^*} > 0, \quad \frac{\partial \dot{w}}{\partial \eta^*} < 0.$$

1. **The wage effect of immigration is weaker (i.e., less negative) the easier it is to substitute labor and capital.** If labor and capital are easily substitutable, the effective magnitude of the immigration-induced supply shock is smaller for any particular immigrant influx. As a result, the adverse wage effect is weaker.

2. **The wage effect of immigration is weaker the more "important" labor is in the production process.** This rule is most obvious in the original Cobb-Douglas results presented in equation (3a). If labor were "unimportant", even a relatively small immigrant supply shock would have a disproportionately large effect.

3. **The wage effect of immigration is weaker the more elastic the supply of capital.** Post-immigration adjustments in the capital stock attenuate the initial wage impact. The easier it is for such capital adjustments to take place, the weaker will be the wage effect of immigration.

4. **The wage effect of immigration is stronger the more elastic product demand.** The scale effect is smaller the greater the elasticity of product demand because consumers would then substantially cut back their demand for the domestic good as the price rises. The smaller the scale effect, the larger the immigration-induced real wage cut.[20]

[20] Although the derivatives summarized in equation (24) were calculated using the real wage elasticity that is measured relative to the price of the domestic product, the same qualitative rules can be obtained by using the wage elasticity that is measured relative to the numeraire, as long as the restriction in equation (17) is satisfied.

These rules were derived under the condition of product market neutrality. In the absence of neutrality, the immigration context adds an interesting fifth rule:[21]

5. **The wage effect of immigration is weaker the greater the impact of immigration on the size of the consumer base relative to its impact on the size of the workforce.** The adverse wage impact of immigration will obviously be much weaker if there are "few" workers and "many" consumers.

An Extension

The model summarized above includes two important restrictions. First, the product demand function for the domestic good q does not depend on the consumer's income; second, the price of good y is fixed (so that the foreign export supply curve for y is perfectly elastic). I now extend the framework in a way that relaxes both of these assumptions simultaneously. It is convenient to begin by modeling consumer demand for good y. Suppose the typical consumer's demand function can be written as:

$$y_j = h_j \, p_y^{-\frac{1}{\tau}} \, p^{\frac{1}{\tau}-1} Z_j \,, \tag{25}$$

where h_j is a person-specific shifter detailing a consumer's relative preference for good y; p_y is the price of good y; p continues to be the price of good q, and Z_j is the consumer's income. The consumer's income is equal to w if he is a worker in the domestic economy; r is

[21] The partial derivative for this fifth rule is $\partial \dot{w} / \partial \phi > 0$. Note, however, that if $\phi \neq 1$, the partial derivatives for the other rules will contain an additional term. The sign of this term will generally depend on whether ϕ is less than or greater than 1.

he is a capitalist in the domestic economy; and *x* if he is a consumer in the "rest of the "world." I will assume that the weighting factor *h* may differ between domestic and foreign consumers.

The specification in (25) implies that τ is the inverse price elasticity of demand for good *y*. Further, the demand function builds in two properties: First, consumer demand is derived from homothetic preferences (implying that the income elasticity is unity). Second, the demand function is homogeneous of degree zero (so that the three elasticities defined in (25) add up to zero).

The total demand for the imported good is given by:

(26) $$Y = p_y^{-\frac{1}{\tau}} p^{\frac{1}{\tau}-1}\left(h_D(C_L w + C_K r) + h_X C_X x\right).$$

Let $W_y = h_D(C_L w + C_K r) + h_X C_X x$, the "effective" wealth that determines aggregate demand for the imported good. The inverse demand function for *Y* can then be written as:

(27) $$p_y = Y^{-\tau} p^{1-\tau} W_y^{\tau}.$$

The aggregate supply curve for *Y* is given by:

(28) $$p_y = Y^{\varphi},$$

where φ is the inverse elasticity of supply. The equilibrium price of good *y* is then determined by the simultaneous solution of equations (27) and (28).

The aggregate demand curve for *Q* when there are wealth effects can be derived in an analogous fashion and is given by:

$$(29) \qquad Q = p^{-\frac{1}{\eta}} p_y^{\frac{1}{\eta}-1} W_q,$$

where $W_q = g_D(C_L w + C_K r) + g_X C_X x$, and measures the effective wealth that determines aggregate demand for the domestic good. As before, the demand function in (29) is derived from homothetic preferences and is homogeneous of degree zero. By substituting in the expression for the equilibrium price of good *y*, it is possible to solve for the inverse aggregate demand function for the domestic good:

$$(30) \qquad p = Q^{-\hat{\eta}} W_q^{\hat{\eta}} W_y^{\hat{\varphi}},$$

where $\hat{\eta}$ and $\hat{\varphi}$ are "rescaled" values of the parameters η and φ, respectively, and depend on the various elasticities of the model.[22] Both of these rescaled elasticities must lie between zero and one. In addition, $\hat{\eta} = \eta$ and $\hat{\varphi} = 0$ when the supply curve for the imported good is perfectly elastic.

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[22] In particular, the rescaled elasticities are defined as $\hat{\eta} = \dfrac{\eta}{1-\varphi^*(1-\tau)(1-\eta)}$ and $\hat{\varphi} = \dfrac{\varphi^*\tau(1-\eta)}{1-\varphi^*(1-\tau)(1-\eta)}$, where $\varphi^* = \dfrac{\varphi}{\varphi+\tau}$.

It is instructive to compare equation (30) with equation (10), the analogous aggregate demand function in the model that ignored wealth effects and assumed that the supply of *Y* was perfectly elastic. In the simpler model, the inverse demand function for *Q* was given by $p = Q^{-\eta} C^{\eta}$, and the shifter was simply the (weighted) number of consumers who purchased the domestic good. Even if the supply curve of *Y* were perfectly elastic (implying $\hat{\varphi}$ = 0), the shifter in equation (30) differs because it depends not only on the number of consumers, but also on the average income of consumers. Further, the possibility that the export supply curve is upward sloping implies that the demand for *Q* also depends on the effective wealth of the consumer base for *Y*.

The presence of wealth effects implies that the impact of immigration on wages in the domestic labor market will now depend on: (a) how immigration changes the size of the consumer base for each of the two goods; and (b) how immigration changes the average income of the consumer base. The first of these effects, of course, is related to the product market neutrality assumption presented earlier. Suppose that the pay rate to each of the groups that make up the consumer base (i.e., *w*, *r*, and *x*) is held constant. The percent change in the size of the consumer base for the domestic good can then be defined by:

(31) $$\phi_q = \left. \frac{d \log W_q}{d \log L} \right|_{w,r,x} = \varepsilon_D^q s_L \frac{d \log C_L}{d \log L} + \varepsilon_D^q s_K \frac{d \log C_K}{d \log L} + (1 - \varepsilon_D^q) \frac{d \log C_X}{d \log L},$$

where $\varepsilon_D^q = 1-(g_X C_X x / W_q)$, the share of total expenditures in good q that is attributable to domestic consumers. The elasticity ϕ_q, of course, is the counterpart of the elasticity ϕ defined in the simpler model above.

The impact of immigration on the size of the consumer base for the imported good is:

(32) $$\phi_y = \left.\frac{d\log W_y}{d\log L}\right|_{w,r,x} = \varepsilon_D^y s_L \frac{d\log C_L}{d\log L} + \varepsilon_D^y s_K \frac{d\log C_K}{d\log L} + (1-\varepsilon_D^y)\frac{d\log C_X}{d\log L},$$

where $\varepsilon_D^y = 1-(h_X C_X x / W_y)$, the share of total expenditures in good y that is attributable to domestic consumers. If much of the output of the domestic good is consumed domestically, while much of the output of the imported good is consumed abroad, it would be reasonable to expect that the elasticity ϕ_q would be relatively large (perhaps nearing one), while the elasticity ϕ_y would be relatively small (perhaps nearing zero).

There's one final issue that needs to be addressed before I can calculate the impact of immigration on the wage in the domestic labor market. It is clear from the definitions of W_q and W_y that the immigration-induced wealth effect will also depend on what happens to x, the pay rate in the foreign economy. For simplicity, I will assume that immigration (though it may be large relative to the size of the domestic workforce) is small relative to the size of the workforce in the rest of the world. Hence I ignore any potential effects on consumer demand (and the domestic labor market) through the change in the level of foreign income x.

The mathematical appendix presents the complete derivation of this model. The resulting expressions are more complex because of the rapidly exploding number of elasticities (and income shares) required to describe all the various feedback effects in the model. To simplify the analysis, I consider the case of a Cobb-Douglas production function. Further, I restrict the discussion to the impact of immigration on the real wage (w/p). Despite the additional complexity, some unambiguous results emerge. In particular, the real wage elasticity in the expanded model is given by:

$$\frac{d\log(w/p)}{d\log L} = \frac{-\lambda(1-\hat{\eta}\varepsilon_D^q - \hat{\varphi}\varepsilon_D^y)s_K}{\Delta} + \frac{\hat{\eta}(\phi_q - 1) + \hat{\varphi}\phi_y}{\Delta}, \tag{33}$$

where $\Delta = (1+\lambda)[1-\hat{\eta}\varepsilon_D^q s_L - \hat{\varphi}\varepsilon_D^y s_L] - [(1-\hat{\eta}) + \hat{\eta}\lambda\varepsilon_D^q + \hat{\varphi}\lambda\varepsilon_D^y]s_K$.

It is instructive to compare the real wage elasticity in equation (33) with the analogous expression in (22), which ignored wealth effects and assumed a perfectly elastic supply of the imported good y.[23] For simplicity, suppose that product market neutrality holds in both models, so that the second term in both equation (22) and equation (33) vanishes.[24] It is easy to see that the numerator of (33) contains the additional term $\lambda(\hat{\eta}\varepsilon_D^q + \hat{\varphi}\varepsilon_D^y)s_K$. This additional term represents two distinct scale effects induced by the impact of the changing wealth of consumers on product demand. Both of these scale effects

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[23] In the special case where the supply curve of the imported good is not perfectly elastic and there are no wealth effects, all the terms involving the share ε in equation (33) would drop out.

[24] The "generalized" form of product market neutrality required in this more general model is given by $\hat{\eta}(\phi_q - 1) + \hat{\varphi}\phi_y = 0$.

are positive, so that they weaken the adverse labor market impact of immigration as long as capital has not fully adjusted.

The first wealth effect (the term multiplied by $\hat{\eta}$) arises because immigration necessarily increases "average" income in the domestic economy. In other words, as is typical in this type of model, the losses incurred by workers are more than made up by the gains accruing to capitalists (Borjas, 1995). The wealth effect generated by the higher average income increases demand for the domestically produced good, increases the demand for labor, and helps to attenuate the adverse labor market impact of immigration. The second wealth effect (the term multiplied by $\hat{\varphi}$) arises because immigration also generates increased demand for the imported good *y*. If the supply curve of the imported good is not perfectly elastic, however, the wealth effect increases the price of the imported good, encouraging consumers to switch back to the domestic good, and further attenuates the adverse impact of immigration on the domestic wage.

Regardless of the size of all these feedback effects, however, equation (33) implies three unambiguous results as long as there is "generalized" product market neutrality:

1. The wage impact of immigration will be zero in the long run (i.e., when $\lambda = 0$).

2. As shown in the mathematical appendix, the real wage elasticity in (33) must be negative as long as there is incomplete capital adjustment.[25] In other words, the scale effects generated by the immigration-induced changes in consumer demand can never be

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[25] The proof uses the fact that the rescaled elasticities $\hat{\eta}$ and $\hat{\varphi}$ are both positive and less than one. It is easy to verify that their sum is also less than one. To show that the sign of the numerator in (33) must be negative, simply evaluate the numerator at the point where the income shares in the expression take on the value of 1, so that the scale effects are at their maximum value. Using the same numerical properties of the rescaled elasticities, it can also be shown that the denominator must be positive.

sufficiently strong to reverse the direct adverse impact of immigration in the domestic labor market.

3. The short-run effect of immigration is given by:

(34) $$\left.\frac{d\log(w/p)}{d\log L}\right|_{\substack{\lambda=\infty \\ \delta=0}} = -s_K\,.$$

Remarkably, we have come full circle to the beginning of the analysis (see equation (2a)). The short-run real wage elasticity of immigration is still -0.3, even after the model accounts for all the feedback possibilities introduced by wealth effects in the product market and inelastic supply of the imported good.[26]

IV. Heterogeneous Labor

It turns out that the analytics—and closed-form solutions—of the homogeneous labor model are extremely useful and applicable when the analysis is generalized to include heterogeneous labor. There are two ways in which heterogeneous labor can be incorporated into the model. First, the workforce can be composed of different skill groups (e.g., education groups). Second, it is possible that immigrants and natives in the same skill group are not perfect substitutes in the production process. Both of these possibilities can

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[26] The real wage can also be defined as the ratio of w to a price index $\overline{p} = p_y^{1-\varepsilon_D^q}\, p^{\varepsilon_D^q}$., where the consumption shares in the exponents are treated as constants. A sufficient condition for the short-run real wage elasticity to be negative is that goods q and y be gross substitutes (i.e., $\tau < 1$).

be easily addressed by using the nested CES framework that has become popular in the immigration literature since its introduction in Borjas (2003).[27]

It is convenient to think of the labor input L defined in the previous section as a labor aggregate—an aggregation of different types of workers. The Armington aggregator that combines different labor inputs is given by:

(35) $$L = [\theta_1 L_1^{\beta} + \theta_2 L_2^{\beta}]^{1/\beta},$$

where L_i gives the number of workers in group i; the elasticity of substitution between groups 1 and 2 is defined by $\sigma_{12} = 1/(1 - \beta)$, with $\beta \leq 1$; and $\theta_1 + \theta_2 = 1$. Although the exposition uses two different labor inputs, it will be evident that all of the results extend to any number of inputs.

To fix ideas, think of the inputs L_1 and L_2 as representing different skill groups. For example, they may represent different education and/or different age groups. Initially, I assume that natives and immigrants within each of the skill groups are perfect substitutes.

Immigrants can shift the supply of either of the two groups. Let $m_i = dL_i/L_i$ give the immigration-induced percent supply shift for group i. It is easy to show that the percent shift in the aggregate labor input is given by:

(36) $$d \log L = \frac{s_1}{s_L} m_1 + \frac{s_2}{s_L} m_2 = \bar{m}.$$

[27] Although restrictive, the use of the nested CES framework greatly simplifies the analytics of the problem and leads to a greater understanding of the underlying economics. Borjas (1999) discusses some of the implications when the different labor inputs are not nested in the production technology.

where s_i is the share of labor income accruing to group i, and $s_L = s_1 + s_2$.

The modeling of product demand in a market with heterogeneous labor requires particular attention as workers differ in their productivity and will inevitably have different resources when they enter the product market. Nevertheless, it is easier to grasp the intuition of the underlying economics by considering the simpler model that ignores wealth effects and assumes that the supply of the imported good is perfectly elastic. For expositional convenience, I will also assume that there is product market neutrality. It is important to emphasize, however, that all of the results presented below carry through to the more general model with non-neutrality, wealth effects, and an upward sloping foreign export supply curve.

In the simplest homogeneous labor model derived in the previous section, the inverse product demand function was represented by $p = C^{\eta}Q^{-\eta}$, where the effective number of consumers for the domestic good is $C = g_D(C_L + C_K) + g_x C_X$. I will use this same market demand function in the heterogeneous labor model. The fact that workers are heterogeneous—and that this heterogeneity affects aggregate demand—can be easily captured by assuming that the shifter $C_L = kL$, so that the number of worker consumers is proportional to the number of efficiency-units adjusted workers. Note that despite the algebraic similarity, the use of this demand function in the current context builds in a very different assumption about the product demand exhibited by different groups of workers. In particular, even though all consumers are assumed to have the same elasticity of product demand, the level of the demand curve differs across workers. The shifter in the inverse product demand function depends on the efficiency units-adjusted number of workers.

Those workers who are more productive and have higher wages will "count" proportionately more in the aggregation. By construction, therefore, the immigration-induced shift in the product demand curve depends on the skill composition of the immigrant population.

The model, therefore, consists of the inverse product market demand function in equation (10), the aggregate production function in equation (11), the inverse supply curve of capital in equation (12), and the aggregator function given by equation (35). The condition that the wage of input *i* equals the group's value of marginal product is given by:

$$w_i = \left[(1-\alpha) C^{\eta} Q^{1-\delta-\eta} L^{\delta-1}\right] \theta_i L^{1-\beta} L_i^{\beta-1}. \tag{37}$$

It is obvious that the marginal productivity condition for the price of capital is identical to that found in the homogeneous labor model in equation (13b). Equally important, the marginal productivity condition for skill group *i* in equation (37) is very similar to that in the homogeneous labor model in equation (13a). In fact, the bracketed term in (37) is *identical* to the value of marginal product of labor in the homogeneous labor case. The fact that there are now two different skill groups simply adds the multiplicative term that appears to the right of the bracket.

Let w^* be the ratio of the bracketed term in (37) to the price level *p*. Hence w^* is the *real* wage paid to the "average" efficiency-unit adjusted worker. By using the supply shift defined in (36), it is easy to show that the effect of immigration on the real wage of group *i* is given by:

$$\text{(38)} \qquad \begin{aligned} d\log w_i &= d\log w^* + (1-\beta)\, d\log L + (\beta - 1)\, d\log L_i, \\ &= d\log w^* + (1-\beta)(\bar{m} - m_i). \end{aligned}$$

Equation (38) has a number of important properties. Suppose, for instance, that we are interested in the distributional effect of immigration—that is, the impact of immigration on the relative wage of the two skill groups. The relative wage effect is given by:

$$\text{(39)} \qquad d\log w_1 - d\log w_2 = (\beta - 1)(m_1 - m_2) = -\frac{1}{\sigma_{12}}(m_1 - m_2).$$

Equation (39) establishes an important property of the model: the distributional wage effect of immigration depends only on the elasticity of substitution between the two groups and is proportional to the magnitude of the supply shift differential between the groups. If the two groups are perfect substitutes, immigration has no relative wage effect. If the two groups are imperfect substitutes, the group that experiences the larger supply shock will *always* experience a decline in its relative wage. It is worth emphasizing that none of the variables that played a crucial role in the homogeneous labor model (e.g., η, λ, σ, and s_i) help determine the distributional impact of immigration.

The simplicity of equation (39)—and, in particular, the property that relative wage effects are proportional to relative supply shifts—suggests that one must be extremely careful when defining skill groups in empirical work and that there are valid reasons for skepticism when evaluating this type of empirical evidence. After all, it is easy to manipulate results by defining skill groups in ways that either accentuate the relative supply shift or that mask it. For instance, if the skill groups are defined in a way that immigration has a proportionately similar increase on the size of every group, it will necessarily follow that immigration has no distributional impact.[28]

In addition to any distributional effects, immigration will also have an impact on the real wage level of all workers, and this level effect is determined by the underlying parameters of the factor demand framework. In particular, define the average real wage effect by $d\log\overline{w} = (s_1\, d\log w_1 + s_2\, d\log w_2)/s_L$.[29] Equation (38) then implies that:

$$(40) \qquad d\log\overline{w} = d\log w^* = \frac{-\lambda(1-\delta)s_K}{(1+\lambda-\delta)-(1-\delta-\eta)s_K}\,\overline{m},$$

which is *identical* to the expression in equation (22) that defines the real wage elasticity in the homogeneous labor model. In other words, regardless of how different workers interact in production, equation (40) shows that the impact of immigration on the average real wage will be negative as long as the capital stock has not fully adjusted to the supply shift.

28 This point was first illustrated in Borjas, Freeman, and Katz (1997). Their simulations show that the wage effect of immigration in the United States is much greater when one compares high school dropouts to the rest of the workforce than when one compares high school "equivalents" to the rest of the workforce. The difference arises because, as typically defined, "high school equivalents" include both high school dropouts and high school graduates. There has been a substantial influx of foreign-born high school dropouts to the United States, but the numerical importance of this supply shift is obviously very different if the competing group consists only of native high school dropouts or if it also includes the millions of native high school graduates.

29 To avoid confusion, I am defining the real wage in terms of the price of the domestically produced product. The argument would be identical if the wage were defined in terms of the numeraire.

The model helps to clarify an important source of potential confusion: the key variables in the homogeneous labor demand framework (η, λ, σ, s_i, and ϕ) determine the real wage elasticity of immigration. For instance, in a Cobb-Douglas world with perfectly elastic product demand, the model predicts that the wage elasticity of immigration will lie between -0.3 (in the short run) and 0.0 (in the long run). These theoretical predictions are not affected *at all* by the way in which various skill groups interact in the labor market. Most important, it does not matter how complementary different groups may be. In an important sense, the wage level effect has been "pre-set" numerically regardless of the type of worker interactions that occur in the production process.

The only role played by the degree of substitutability among the various skill groups is to "place" the wage effect for each of the groups around this pre-determined wage level effect. For instance, in the short-run Cobb-Douglas world, the elasticity of substitution σ_{12} will place one group above -0.3 and another group below -0.3. For a given elasticity of substitution, the specific deviations from -0.3 will depend on: a) the disparity in the immigrant supply shocks experienced by the two groups; and b) the relative income shares of the groups, since the average of the two wage effects has to be identically equal to -0.3. Put differently, *the constraints imposed by the factor demand framework greatly restrain the types of immigration wage effects that can possibly be estimated by any data.*

It is worth emphasizing the same point in a different way: the literature now contains a number of simulations claiming that data analysis based on structural factor demand models implies that the impact of immigration on the wage of the average worker is *x* percent, or that the impact of immigration on the average wage of workers in a particular skill group is *y* percent. These simulations typically use a Cobb-Douglas aggregate production function. The use of this production function is definitely not innocuous. As we have seen, the Cobb-Douglas functional form builds in *numerically* what the average wage effect of immigration must be. In other words, the wage level effects reported by these studies have nothing to do with the underlying data; they are simply "spewing out" the constraints imposed by factor demand theory.

Imperfect Substitution Between Immigrants and Natives

It is easy to expand the model to consider the implications of a recent development in the literature. Some recent studies (Ottaviano and Peri, 2005; Manacorda, Manning, and Wadsworth, 2006; Card, 2009) have argued that immigrants and natives within a skill group are imperfect substitutes—and that the resulting complementarities could greatly attenuate the adverse wage impact of immigration on the pre-existing workforce.

It is important to emphasize that the type of imperfect substitution proposed by these studies is not between immigrants and natives on the aggregate. This type of substitution would be easily handled by the model presented above, since the Armington aggregator in (35) that defines the labor input L can be reinterpreted by letting L_1 represent a homogeneous group of native workers, and L_2 represent a group of immigrant workers. As we have seen, imperfect substitution of this type can only help us understand distributional differences between immigrants and natives. The determinants of the impact of immigration on the average wage level are exactly the same as they were in the homogeneous labor model.

Instead, the recent studies argue that imperfect substitution between immigrants and natives exists even among workers who belong to the *same* skill group. As I will show shortly, however, this type of imperfect substitution also does not add anything to our understanding of the determinants of the impact of immigration on average wages. Imperfect substitution within skill groups can only generate additional distributional effects.

The potential presence of imperfect substitution between equally skilled natives and immigrants implies that there exists yet another aggregator function that sums up the contribution of immigrants and natives for skill group *i*. Let the Armington aggregator be:

(41) $$L_i = [\rho_N N_i^{\gamma} + \rho_F F_i^{\gamma}]^{1/\gamma},$$

where N_i and F_i give the number of native and immigrant workers in skill group *i*, respectively; the elasticity of substitution between native and immigrant workers is σ_{NF} = $1/(1 - \gamma)$, with $\gamma \leq 1$; and $\rho_N + \rho_F = 1$.

There are now separate marginal productivity conditions for native and immigrant workers. For skill group *i*, these conditions can be written as:

(42a) $$w_i^N = \left[(1-\alpha)C^{\eta} Q^{1-\delta-\eta} L^{\delta-1}\right](\theta_i L^{1-\beta} L_i^{\beta-1})(\rho_N L_i^{1-\gamma} N_i^{\gamma-1}),$$

(42b) $$w_i^M = \left[(1-\alpha)C^{\eta} Q^{1-\delta-\eta} L^{\delta-1}\right](\theta_i L^{1-\beta} L_i^{\beta-1})(\rho_F L_i^{1-\gamma} F_i^{\gamma-1}),$$

where w_i^N and w_i^F give the wage of native and immigrant workers in skill group *i*, respectively.

By comparing equations (42a) and (42b) with equation (37), it is obvious that the addition of within-group imperfect substitution simply adds yet another multiplicative term to the marginal productivity conditions. As before, the bracketed term represents the "average" wage level in the economy, aggregated across all skill groups. This is the wage level determined by the factor demand theory parameters introduced in Section III. The product of this bracketed term and $\theta_i L^{1-\beta} L_i^{\beta-1}$ gives equation (37), the marginal productivity condition that determines the wage for the average worker in skill group *i*. The introduction of within-group imperfect substitution simply adds another multiplicative term. Put differently, the presence of within-group immigrant-native complementarities cannot play any role in determining how immigration affects the average wage level in the labor market or the average wage level of a skill group.

The multiplicative separability property allows us to easily assess the potential importance of immigrant-native complementarities in determining the wage structure. By differentiating equations (42a) and (42b), it is easy to calculate the impact of an immigration-induced supply shift on the relative wage of native workers and immigrant workers in group *i*:

(43) $$d \log w_i^N = d \log w^* + (1-\beta)[d \log L - d \log L_i] + (1-\gamma)[d \log L_i - d \log N_i].$$

(44) $$d \log w_i^M = d \log w^* + (1-\beta)[d \log L - d \log L_i] + (1-\gamma)[d \log L_i - d \log F_i].$$

Suppose that there is an immigrant supply shock that changes the number of immigrants in each of the skill groups; by assumption, the number of native workers remains fixed. Further, define the immigration-induced percent change in the efficiency-adjusted size of the workforce by $\bar{m}$ and the percent change in the size of the efficiency units-adjusted workforce in skill group *i* by m_i. By using the aggregator functions, it is easy to show that these average supply shifts are given by:

(45) $$d\log L = \bar{m} = \frac{s_1^F}{s_L} f_1 + \frac{s_2^F}{s_L} f_2,$$

(46) $$d\log L_i = m_i = \frac{s_i^F}{s_i} f_i,$$

where $f_i = dF_i/F_i$, the percent increase in the size of the foreign-born workforce in skill group *i*; s_L is labor's share of income; s_i is the share of income accruing to group *i*; and s_i^F is the share of income accruing to immigrants in group *i*.

It is instructive to first examine how the immigration-induced supply shift changes the wage of natives in a particular skill group. The definition of the supply shifts implies that equation (43) can be rewritten as:

(47) $$d\log w_i^N = d\log w^* + (1-\beta)[\bar{m} - m_i] + (1-\gamma)\frac{s_i^F}{s_i} f_i.$$

It is instructive to think of the wage effect summarized in equation (47) in terms of the three levels of the nested CES production function. The first term shows how immigration affects the average wage level in the labor market. It is this wage effect that depends on the parameters described in Marshall's rules of derived demand (with the addition of the elasticity relating the size of the consumer base to the influx of immigrants).

The second term introduces the distributional effects that arise because different workers belong to different skill groups—e.g., some workers will be high school dropouts while others will be college graduates. As I showed in the previous section, the possibility that workers in different skill groups are not perfect substitutes ($\beta \neq 1$) does not change the effect of immigration on the average worker. The productive interaction among the various skill groups simply places the wage impact experienced by the various groups around the "pre-determined" wage level effect.

Finally, the last term in equation (47) accounts for potential complementarities between equally skilled immigrants and natives, and will lead to a positive wage effect on natives as long as immigrants and natives are not perfect substitutes ($\gamma \neq 1$). It is important to emphasize, however, that the presence of these within-group complementarities does not change the effect of immigration either on the average wage in the labor market or on the average wage of workers in skill group *i*. The potential complementarities simply "place" the wage effect on natives and pre-existing immigrants who belong to the same skill group around the pre-determined mean wage effect for workers in that skill group.

Equation (47) shows that the presence of complementarities will increase the wage of natives by the amount $(1/\sigma_{NF})(s_i^F / s_i) f_i$, where σ_{NF} is the elasticity of substitution between immigrants and natives. This simple expression can be used to get a sense of the numerical importance of these presumed complementarities.

Suppose, for example, that a new immigrant influx doubles the size of the pre-existing immigrant workforce in the average skill group, so that f_i = 1.0. Suppose further that immigrants in this average skill group make up around 10 percent of the total workforce in that skill group. In rough terms, this would imply that s_i^F / s_i is around 0.1.[30] In a series of papers, Ottaviano and Peri (2005, 2008) have argued that within-group immigrant-native complementarities are quantitatively important. However, their estimate of the elasticity of substitution σ_{NF} has risen substantially over time, from 5.5 in their initial paper to 20 in their current iteration.[31] Suppose that indeed the elasticity of substitution σ_{NF} is 20. It is then evident that a doubling of the immigrant workforce in a world where immigrants make up around 10 percent of the average skill group's workforce would increase the native wage by only around 0.5 percent (or $(1/20) \times 0.1 \times 1.0$). In other words, an elasticity of substitution of around 20 is operationally equivalent to a world in which immigrants and natives are perfect substitutes. Again, the implications of the factor demand framework provide a significant constraint on the types of arguments that can be made about the wage effect of immigration and can be used to check the plausibility of various empirical claims in the immigration literature.

Equations (43) and (44) also imply that the presence of imperfect substitution between equally skilled immigrants and natives would have important consequences for

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[30] The estimate of the relative income share accruing to immigrants in a particular skill group is likely to be substantially smaller if immigrants earn less than equally skilled natives.

[31] As pointed out by Borjas, Grogger, and Hanson (2008), the original Ottaviano-Peri (2005) study contained several data flaws that contaminated the analysis. Most conspicuous, Ottaviano and Peri classified millions of native-born currently enrolled high school juniors and seniors as "high school dropouts." The simple exclusion of these students turns their estimate of the inverse elasticity of substitution from (1/5.5) to near 0, both numerically and statistically. The glaring nature of the data problems have led Ottaviano and Peri (2008) to admit that "the original Ottaviano and Peri estimate of σ_{IMMI} [the inverse elasticity of substitution between immigrants and natives] was probably too large…" In their most recent work, Ottaviano-Peri correct for some (though not all) of the problems pointed out by Borjas, Grogger, and Hanson. It is worth noting that Borjas, Grogger, and Hanson conclude that the data do not allow the rejection of the hypothesis that equally skilled immigrants and natives in the United States are perfect substitutes.

within-group inequality. In particular, the relative wage effect of new immigrants on the relative wage of pre-existing immigrants and natives can be written as:

(48)
$$d\log w_i^M - d\log w_i^N = -(1-\gamma) f_i = -\frac{1}{\sigma_{NF}} f_i .$$

If the elasticity of substitution between immigrants and natives is on the order of 20 and if new immigration doubles the size of the foreign-born workforce for the average skill group, the wage gap between immigrants and natives would grow by 5 percentage points. In other words, the presence of complementarities in a group of equally skilled natives and immigrants would necessarily increase the wage gap between the two groups, and this wage increase would be numerically sizable even if the elasticity of substitution is relatively large. Put differently, factor demand theory implies that new immigration will lead to a substantial growth in immigrant-native inequality if the two groups are imperfect substitutes. As a result, empirical claims of significant within-group complementarity should be interpreted cautiously unless there is complementary evidence of a concurrent increase in immigrant-native wage dispersion.

V. Summary

This paper presents the simple analytics that underlie the study of the wage effects of immigration. A crucial insight of the factor demand framework is that the effect of immigration on the average wage level depends on factors that are completely different than the factors that determine the distributional effect of immigration (i.e., the effect of immigration on the relative wage of different groups of workers). In particular, the parameters that Marshall identified in his famous rules of derived demand (i.e., the elasticity of substitution between labor and capital, the supply elasticity of capital, the elasticity of product demand, and labor's share of income) jointly determine the impact of immigration on the average wage level. In addition, the immigration context shows the importance of an additional parameter: the impact of immigration on the size of the consumer base relative to its impact on the size of the workforce. Once this average wage effect is determined, the way in which various skill groups of workers interact and the extent to which immigration shifts the supply of some groups by more than others places the various groups of workers around the pre-determined aggregate wage effect.

The simplicity of the factor demand framework also leads to closed-form solutions of the wage effect of immigration. As a result, it is quite easy to use the resulting equations to conduct back-of-the-envelope calculations of what the wage effect of immigration could conceivably be under a large number of potential scenarios. In effect, the constraints imposed by the theory of labor demand can be used to assess the plausibility of the many contradictory claims that are often made about how immigration affects the wage structure in sending and receiving countries.

The theoretical framework presented in this paper can be extended in a number of ways. For instance, the domestic economy can produce different types of goods (some may be labor-intensive and some may be capital-intensive). An immigration-induced supply shift would necessarily induce flows of resources among the various sectors. It would be of interest to determine whether the aggregate wage impact of immigration (i.e., the impact that averages out the wage effect across sectors) is affected by the leveraging possibilities introduced by the multi-sector framework.

The paper also raises questions for empirical research. The theoretical framework, after all, highlights the importance of determining how immigration changes the relative number (and wealth) of consumers. The theory clearly demonstrates that an imbalance between the impact of immigration on the size of the consumer base and its impact on the size of the workforce can generate long-run wage effects. It seems, therefore, that the consumption behavior of immigrants is a topic ripe for empirical analysis.

Finally, the study highlights an important disconnect between factor demand theory and some of the empirical work in the literature. The theory unambiguously predicts that the short run wage effect of immigration will be negative and numerically important, even after accounting for a wide array of immigration-induced feedback and scale effects. If one is to believe the empirical claim that immigration wage effects are negligible *even in the short run*, the theoretical implications of factor demand theory need to be dismissed and the entire apparatus thrown by the wayside. We are then left without a framework for understanding or predicting how immigration influences labor market conditions in sending and receiving countries.

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Mathematical Appendix

1. Derivation of Equation (14)

The mathematical derivation is greatly simplified by adapting the approach in Kennan (1998). First, note that the value of marginal product condition for labor can be written as:

(A1) $$\frac{w}{p} = (1-\alpha)Q^{1-\delta}L^{\delta-1}.$$

By substituting the inverse product demand curve in (A1), it is easy to show that:

(A2) $$\log w = \eta \log C + \log(1 - \alpha) + (1 - \eta - \delta) \log Q + (\delta - 1) \log L.$$

Differentiating with respect to the immigrant supply shift yields:

(A3) $$\frac{d\log w}{d\log L} = \eta\phi + (1-\eta-\delta)\frac{d\log Q}{d\log L} + (\delta-1).$$

Next, note that the ratio of input prices in a CES technology can be written as a simple function of the ratio of input quantities. In particular:

(A4) $$\frac{w}{r} = \frac{(1-\alpha)L^{\delta-1}}{\alpha K^{\delta-1}}.$$

Differentiating this expression, while accounting for the fact that $r = K^{\lambda}$, yields:

(A5) $$\frac{d\log K}{d\log L} = \frac{1}{1+\lambda-\delta}\left(\frac{d\log w}{d\log L} + (1-\delta)\right).$$

Finally, the CES production function implies that:

(A6) $$\frac{d\log Q}{d\log L} = s_K\frac{d\log K}{d\log L} + s_L.$$

Equation (10) can be derived by substituting equations (A5) and (A6) into (A3).

2. The Social Planner Problem

The maximization problem faced by the social planner is:

(A7) $$\text{Max } \Omega = pQ - Mh = C^{\eta}Q^{1-\eta} - Mh.$$

Suppose that there is product market neutrality. Without loss of generality, I can then write $C = L$. The first-order condition that determines the optimal number of immigrants is:

(A8) $$\frac{\partial \Omega}{\partial M} = \eta L^{\eta-1} Q^{1-\eta} + (1-\alpha)(1-\eta) L^{-(1-\delta-\eta)} Q^{1-\delta-\eta} - h = 0.$$

The second order conditions is given by:

(A9) $$\frac{\partial^2 \Omega}{\partial M^2} = -\eta(1-\eta) L^{\eta-2} Q^{1-\eta} s_K - (1-\eta)(1-\delta-\eta)(1-\alpha) L^{\delta+\eta-2} Q^{1-\delta-\eta} s_K < 0.$$

Sufficient conditions for (A9) to hold are $(1 - \eta) > 0$ and $(1 - \delta - \eta) > 0$.

3. The Extended Model

Equation (30) in the text shows that the aggregate demand function for the domestic good (after solving out the equilibrium price of good y) is:

(A10) $$p = Q^{-\hat{\eta}} W_q^{\hat{\eta}} W_y^{\hat{\varphi}},$$

where $W_q = g\,(C_L w + C_K r) + g_X C_X x$, and $W_y = h\,(C_L w + C_K r) + h_X C_X x$. The rescaled elasticities in this aggregate demand function are defined by:

(A11) $$\hat{\eta} = \frac{\eta}{1-\varphi^*(1-\tau)(1-\eta)},$$

(A12) $$\hat{\varphi} = \frac{\varphi^* \tau (1-\eta)}{1-\varphi^*(1-\tau)(1-\eta)},$$

(A13) $$\varphi^* = \frac{\varphi}{\varphi+\tau}.$$

The inverse demand function in equation (A10) only makes economic sense (i.e., price depends negatively on quantity and positively on income) if $\hat{\eta} > 0$. As a result, the denominator of (A11) must be positive. The second-order conditions for the social planner problem in this extended model will be satisfied if $\hat{\eta} < 1$. By using the definition in (A11), it is easy to demonstrate that the restriction that $\hat{\eta} < 1$ also implies that η is less than 1. Using these properties, it then follows that $\hat{\varphi}$ must also lie between 0 and 1. These numerical restrictions will be used below.

Suppose the production function is Cobb-Douglas. Using (A10), one can then differentiate the marginal productivity condition to obtain:

(A14) $$\frac{d \log w}{d \log L} = -\hat{\eta} \frac{d \log Q}{d \log L} + \hat{\eta} \frac{d \log W_q}{d \log L} + \hat{\varphi} \frac{d \log W_y}{d \log L} + \alpha \frac{d \log K}{d \log L} - \alpha.$$

The wage elasticity depends on how the effective wealth variables W_q and W_y change as a result of the immigration-induced supply shift. It is easy to show that:

(A15)
$$\frac{d\log W_q}{d\log L}=\frac{g_D C_L w}{W_q}\frac{d\log C_L}{d\log L}+\frac{g_D C_K r}{W_q}\frac{d\log C_K}{d\log L}+\frac{g_X C_X x}{W_q}\frac{d\log C_X}{d\log L}$$
$$+\frac{g_D C_L w}{W_q}\frac{d\log w}{d\log L}+\frac{g_D C_K r}{W_q}\frac{d\log r}{d\log L},$$

where I assume that immigration is sufficiently "small" to leave average income in the "rest of the world" (x) unchanged. The assumption that the weights g in the demand function for the domestic good (and h for the imported good) are the same for domestic labor and capital allows equation (A15) to be rewritten in terms of income shares. In particular, I assume that the share of consumption attributable to domestic labor or capital equals their share of income. Hence:

(A16)
$$\frac{d\log W_q}{d\log L}=\varepsilon_D^q s_L\frac{d\log C_L}{d\log L}+\varepsilon_D^q s_K\frac{d\log C_K}{d\log L}+(1-\varepsilon_D^q)\frac{d\log C_X}{d\log L}$$
$$+\varepsilon_D^q s_L\frac{d\log w}{d\log L}+\varepsilon_D^q s_K\frac{d\log r}{d\log L}.$$

Note that the first three terms of (A16) define the elasticity ϕ_q in equation (31).

It follows that the change in the effective wealth determining aggregate demand for good y is:

(A17)
$$\frac{d\log W_q}{d\log L}=\varepsilon_D^y s_L\frac{d\log C_L}{d\log L}+\varepsilon_D^y s_K\frac{d\log C_K}{d\log L}+(1-\varepsilon_D^y)\frac{d\log C_X}{d\log L}$$
$$+\varepsilon_D^y s_L\frac{d\log w}{d\log L}+\varepsilon_D^y s_K\frac{d\log r}{d\log L}.$$

The first three terms of (A17) define the elasticity ϕ_y in equation (32).

The substitution of equations (A5), (A6), (A16) and (A17) into equation (A14) yields:

(A18)
$$\frac{d\log w}{d\log L}=\frac{-\lambda(1-\hat{\eta})s_k+\hat{\eta}\lambda\varepsilon_D^q s_K+\hat{\varphi}\lambda\varepsilon_D^y s_K+(1+\lambda)[\hat{\eta}(\phi_q-1)+\hat{\varphi}\phi_y]}{(1+\lambda)[1-\hat{\eta}\varepsilon_D^q s_L-\hat{\varphi}\varepsilon_D^y s_L]-[(1-\hat{\eta})+\hat{\eta}\lambda\varepsilon_D^q+\hat{\varphi}\lambda\varepsilon_D^y]s_K}.$$

The Cobb-Douglas production function implies that the real wage equals $(w/p)=(1-\alpha)K^{\alpha}L^{-\alpha}$. Using equation (A5), the real wage elasticity can then be written as:

(A19) $$\frac{d\log(w/p)}{d\log L} = \alpha\frac{d\log K}{d\log L} - \alpha = \alpha\frac{\dfrac{d\log w}{d\log L}+1}{1+\lambda} - \alpha,$$

The expression for the real wage elasticity is obtained by substituting (A18) into (A19). This step yields:

(A20) $$\frac{d\log(w/p)}{d\log L} = \frac{-\lambda s_K + \hat{\eta}\lambda\varepsilon_D^q s_K + \hat{\varphi}\lambda\varepsilon_D^y s_K + [\hat{\eta}(\phi_q - 1) + \hat{\varphi}\phi_y]}{(1+\lambda)[1-\hat{\eta}\varepsilon_D^q s_L - \hat{\varphi}\varepsilon_D^y s_L] - [(1-\hat{\eta}) + \hat{\eta}\lambda\varepsilon_D^q + \hat{\varphi}\lambda\varepsilon_D^y]s_K}.$$

Suppose that "generalized" product market neutrality holds. The bracketed term in the numerator of (A20) vanishes.

To show that the real wage elasticity must then be negative, recall that the rescaled elasticities $\hat{\eta}$ and $\hat{\varphi}$ defined in (A11) and (A12) are both greater than zero and less than one. It is straightforward to verify that the sum of these two elasticities is also less than one. The fact that $\hat{\eta}+\hat{\varphi} < 1$ can be used to prove that the numerator of (A20) must be negative. In particular, the numerator equals $-\lambda s_K(1-\hat{\eta}\varepsilon_D^q - \hat{\varphi}\varepsilon_D^y)$. The maximum value that the income shares in this expression can attain is 1. In that case, the numerator equals $-\lambda s_K[1-(\hat{\eta}+\hat{\varphi})]$. But the sum of the two elasticities $\hat{\eta}$ and $\hat{\varphi}$ must be less than one, hence the numerator is negative.

By using analogous arguments it is possible to show that the denominator of (A20) is positive. In particular, evaluate the denominator at $\lambda = 0$. Using the properties of the rescaled elasticities noted above, it is easy to show that the denominator is positive at the lowest possible value of λ. By differentiating the denominator with respect to λ, it is also possible to show that the denominator is a monotonically increasing function of λ.